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ANISOTROPIC STRENGTHENING OF AN ORTHOTROPIC MATERIAL

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The experimental results obtained in [1] in testing thin-walled tubular samples of zirconium alloy under biaxial tension are analyzed. The features of development of elastoplastic deformations and deformation strengthening of the alloy in proportional loading are established. They made it possible to classify the alloy as an orthogonal-anisotropic material and to model the change of different strained states by means of a certain sequence of local slidings along the areas of basic tangential stresses.

1. The results of proportional loading of seven samples at different ratios between the basic stresses — the axial tension (along the cylinder generatrix) σ_z and the circumferential tension σ_φ ($k_\sigma = \sigma_z/\sigma_\varphi$) — are presented in [1]. Figures 1 and 2 illustrate the results of these experiments in the form of strain diagrams $\sigma_z = \sigma_z(\varepsilon_z)$ and $\sigma_\varphi = \sigma_\varphi(\varepsilon_\varphi)$, respectively. The ratio k_σ and the number of the sample tested are given for each of the diagrams.

The data of the tests indicate the presence of initial anisotropy of the elastic deformation and strength properties. To detect elastic anisotropy, the test results at equal biaxial tension $\sigma_z = \sigma_\varphi$ ($k_\sigma = 1$) and biaxial tension $\sigma_z = 0.5\sigma_\varphi$ ($k_\sigma = 0.5$) equivalent to a pure shear stress with superimposed hydrostatic pressure are used. The initial linear sections of the strain diagram are well manifested at these tensions. The following tensions and deformations were taken as initial data from the reference points in such sections: $\sigma_z = 11.14 \cdot 9.81$ MPa, $\sigma_\varphi = 11.18 \cdot 9.81$ MPa, $\varepsilon_z = 0.042\%$, $\varepsilon_\varphi = 0.074\%$ at $k_\sigma = 0.5$; $\sigma_z = 11.59 \cdot 9.81$ MPa $\sigma_\varphi = 27.90 \cdot 9.81$ MPa, $\varepsilon_z = 0.030\%$, $\varepsilon_\varphi = 0.199\%$ at $k_\sigma = 1$. The elastic constants of the generalized Hooke's law [2] were determined:

$$\varepsilon_z = \frac{1}{E_z}\sigma_z - \frac{\nu_{\varphi z}}{E_\varphi}\sigma_\varphi, \qquad \varepsilon_\varphi = \frac{1}{E_\varphi}\sigma_\varphi - \frac{\nu_{z\varphi}}{E_\varphi}\sigma_\varphi \tag{1.1}$$

(E_z, E_φ, $\nu_{z\varphi}$, $\nu_{\varphi z}$ are the elastic constants of the material characterizing its orthotropy and having the sense of Young's modules and Poisson's coefficients).

The elastic constants of the material (at the given initial data) found from relations (1.1) satisfy, with an accuracy of 5%, the equality

$$E_z\nu_{\varphi z} = E_\varphi\nu_{z\varphi}. \tag{1.2}$$

It follows from (1.2) that these constants are the basic ones, and that the samples being tested have three orthogonal planes of elastic symmetry of the material directed along the z axis of the sample, tangentially to the cylinder generatrix φ, and in the direction of the radius r. The directions z, φ, r are also the basic ones.

On the basis of this, for further calculations we finally take

$$E_z = 21000 \cdot 9.81 \text{ MPa}, \quad \nu_{\varphi z} = 0.118, \qquad E_\varphi = 13000 \cdot 9.81 \text{ MPa}, \quad \nu_{z\varphi} = 0.190. \tag{1.3}$$

The values of the plastic strain components (Γ_z and Γ_φ) in axial tension were computed from (1.3). They were determined as the difference between the values of strains ε_z and ε_φ and their elastic constituents

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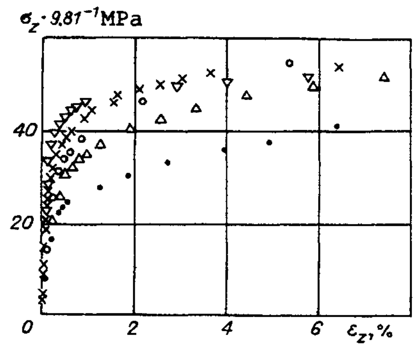

Fig. 1

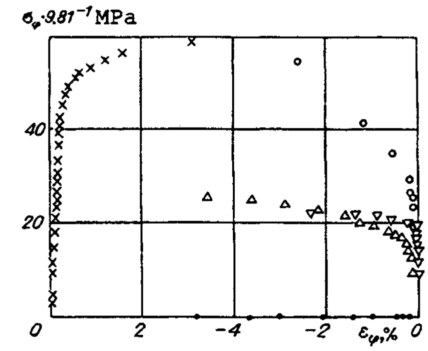

Fig. 2

by Hooke's law (1.1). The data are listed in Table 1. It is seen from Table 1 that the plastic strains at the first five points are such that

$$\Gamma_z \approx -\Gamma_\varphi. \tag{1.4}$$

Consequently, the plastic strain in the radial direction is $\Gamma_r = 0$. This means that the action of the tension $\sigma_z > 0$ causes the appearance and development of pure shear plastic strain (1.4), due, obviously, to only local slidings grouped initially near the direction of action the maximal tangential stress $\tau_{z\varphi}$.

Further increase in the tension σ_z leads to $\Gamma_z > |\Gamma_\varphi|$, the increment of the difference $\Gamma_z - |\Gamma_\varphi|$ from one reference point to another remaining approximately constant up to the 9th point. Then it increases by approximately a factor of 1.8 and remains almost unchanged to the final tension. Such a variation in the relation between the components of plastic strain Γ_z and Γ_φ can be explained by the fact that the sliding areas [3] T_{ij} $(i, j = z, \varphi, r)$, i.e., the areas where the basic tangential stresses act, are "brought into action" in succession.

2. It is customary [4] to determine the yield stresses of anisotropic materials in different stress states by using some tolerance for the largest basic plastic strain because such materials do not have "a unified" strengthening diagram in any kind of generalized coordinates. The value $\Gamma_z = 0.1\%$ is taken as this tolerance for the zirconium alloy under consideration for $\infty \geqslant k_\sigma \geqslant 1$, when the development process of plastic deformation can be considered steady-state, as follows from the strain diagrams (Fig. 1). Moreover, relation (1.4) is valid for this material at these stresses in a small vicinity of such conditional yield stress, i.e., slidings first occur in the area $T_{z\varphi}$. It also follows from the plots in Fig. 1 that the yield stress σ_z^{y} in the pure shear state ($k_\sigma = 2$) is larger by approximately a factor of 1.2 than in axial tension ($k_\sigma = \infty$), for which

$$\sigma_z^{\text{y}} \approx 10 \cdot 9.81 \text{ MPa} \quad (k_\sigma = \infty) \tag{2.1}$$

is obtained. This is the first of the yield stresses in one of the three basic directions. In the two other basic directions, such stresses were determined in the following way.

The stress state at which only the tension σ_φ is different from zero could not be realized because the test facility used is not suitable for compression of samples. Therefore, first the test data of sample No. 6605 loaded in a state close to the pure shear conditions ($k_\sigma \approx 0.5$) were analyzed. This sample demonstrated a noticeable increment of the plastic strain component Γ_φ at tension $\sigma_\varphi \approx 38 \cdot 9.81$ MPa. We assume that the relation between the yield stresses for $k_\sigma = 0.5$ and 0 (at a tolerance $\Gamma_\varphi = 0.1\%$) is the same as the above-mentioned relation between the yield stresses σ_z^{y} for $k_\sigma = 2$ and ∞. As a result,

$$\sigma_\varphi^{\text{y}} \approx 32 \cdot 9.81 \text{ MPa} \quad (k_\sigma = 0). \tag{2.2}$$

TABLE 1

N	$\sigma_z \cdot 9.81$	$\sigma_\varphi \cdot 9.81$	ε_z	$-\varepsilon_\varphi$	Γ_z	$-\Gamma_\varphi$	$\Gamma_z - \lvert\Gamma_\varphi\rvert$	$\Delta(\Gamma_z - \lvert\Gamma_\varphi\rvert)$
	MPa		%					
1	8.47	0	0.107	0.0066	0.066	0.001	0.065	
2	16.42	0	0.250	0.162	0.172	0.147	0.025	−0.040
3	22.25	0	0.413	0.330	0.307	0.310	−0.003	−0.028
4	23.31	0	0.487	0.382	0.376	0.361	0.015	0.018
5	24.34	0	0.533	0.430	0.417	0.408	0.009	−0.006
6	27.55	0	1.300	0.980	1.169	0.955	0.214	0.205
7	30.20	0	1.920	1.413	1.776	1.386	0.390	0.176
8	32.90	0	2.730	2.040	2.573	2.010	0.563	0.173
9	35.76	0	4.013	3.000	3.843	2.968	0.875	0.312
10	37.98	0	4.950	3.660	4.769	3.626	1.143	0.268
11	38.67	0	6.380	4.770	6.196	4.735	1.461	0.318

Note. N is a reference point number in the experiment.

It is known from experimental investigations that the Mises quadratic yield condition [6]

$$H_0(\sigma_z - \sigma_\varphi)^2 + F_0(\sigma_\varphi - \sigma_r)^2 + G_0(\sigma_r - \sigma_z)^2 = 1 \tag{2.3}$$

is valid [5] for an orthotropic material under loadings in its basic directions (H_0, F_0, G_0 are parameters characterizing the initial anisotropy of the material).

It is impossible to determine experimentally the yield stress σ_r^y. Therefore, the yield law associated with condition (2.3) is commonly used. It follows from this law that [7]

$$\frac{1}{(\sigma_r^y)^2} = \frac{1}{(\sigma_\varphi^y)^2} + \frac{1 - 2\nu_{\varphi z}}{(\sigma_z^y)^2}.$$

Substituting the already known material parameters into this relation we obtain

$$\sigma_r^y = 10.77 \cdot 9.81 \text{ MPa} \quad (\sigma_z = \sigma_\varphi = 0). \tag{2.4}$$

Using the yield stresses (in the basic directions) (2.1), (2.2), and (2.4) found in this way, we determine, on the basis of (2.3), the initial anisotropy parameters

$$H_0 = \frac{1}{2}\left[\frac{1}{(\sigma_z^y)^2} + \frac{1}{(\sigma_\varphi^y)^2} - \frac{1}{(\sigma_r^y)^2}\right],$$

$$F_0 = \frac{1}{2}\left[\frac{1}{(\sigma_\varphi^y)^2} + \frac{1}{(\sigma_r^y)^2} - \frac{1}{(\sigma_z^y)^2}\right], \qquad G_0 = \frac{1}{2}\left[\frac{1}{(\sigma_r^y)^2} + \frac{1}{(\sigma_z^y)^2} - \frac{1}{(\sigma_\varphi^y)^2}\right].$$

3. The quadratic yield condition (2.3) characterizes the initial isotropic (but different in each plane of symmetry) loading surface of an orthotropic material (i.e., reflects the so-called normal anisotropy [5]). However, deviations from condition (2.3) are observed in arbitrary loadings [5]. This is also valid for the material under consideration, as shown by the above comparison of the yield stresses σ_z^y for different types of stresses, which do not fit in dependence (2.3). In particular, its strength in biaxial tension is substantially higher than in uniaxial loading.

Such a considerable and stable increase in biaxial strength was also observed for a sheet titanium alloy [8], whose yield stress in equal biaxial tension exceeded the yield stress in uniaxial tension by a factor of about 1.35. A special yield criterion for a plane with anisotropy is proposed in [5] in order to describe such strengthening associated with the material texture. A uniform yield function of an arbitrary fractional degree is introduced instead of the quadratic function. The yield stress in equal biaxial tension and the yield stress

TABLE 2

N	$\sigma_z \cdot 9.81$	$\sigma_\varphi \cdot 9.81$	ε_z	ε_φ	Γ_z	Γ_φ	$\Delta\Gamma_z$	$\Delta\Gamma_\varphi$
	MPa		%					
6	28.076	0	0.973	−0.640	0.8390	−0.6150		
7	28.088	1.265	1.060	−0.662	0.9270	−0.6463	0.0880	−0.0313
8	28.080	4.794	1.080	−0.662	0.9510	−0.6783	0.0730	−0.0320
9	28.136	14.381	1.040	−0.558	0.9190	−0.6731	−0.0320	0.0052
10	28.010	25.233	0.987	−0.478	0.8765	−0.6467	−0.0425	0.0264
11	28.143	32.357	0.947	−0.375	0.8424	−0.5984	−0.0341	0.0483
12	28.057	38.349	0.900	−0.250	0.8012	−0.5196	−0.0412	0.0788
13	27.989	43.143	0.840	−0.156	0.7459	−0.4625	−0.0553	0.0571
14	27.954	45.540	0.793	0.096	0.7032	−0.2290	−0.0427	0.2335
15	27.937	46.733	0.760	0.199	0.6694	−0.1352	−0.0338	0.0938
16	27.948	48.003	0.713	0.346	0.6235	0.0020	−0.0459	0.1372
17	27.989	43.143	0.733	0.338	0.6389	0.0314	0.0154	0.0294
18	27.642	32.424	0.793	0.235	0.6908	0.0106	0.0519	−0.0208
19	27.965	13.982	0.907	−0.037	0.7865	−0.1192	0.0957	−0.1298
20	28.100	8.123	0.987	−0.147	0.8605	−0.1840	0.0740	−0.0648
21	28.014	2.330	1.140	−0.332	1.0087	−0.3235	0.1482	−0.1395
22	28.076	0	1.220	−0.404	1.0863	−0.3786	0.0776	−0.0551

Note. N is a reference point number in the experiment.

in pure shear parallel to the orthotropy axis are taken as initial data. Moreover, the state of the material is characterized by three more material parameters associated with the shape of the yield surface.

A review of different yield criteria for initially anisotropic materials is given in [9]; a search for the criteria most suitable in each particular case is in progress. It is shown [4] that an equation of at least sixth degree with respect to tensions is required to describe the yield surface of conventional (anisotropic) metals determined experimentally, or a similar equation can be represented in the form of anisotropic theory for the second and third invariants of the stress tensor.

It is considered [4] that such an approach is not practical. In all these cases, the character of plastic deformation in its initial phase is not properly taken into account although it is of fundamental importance. As was shown above with the test data of alloy E-110 as an example, the occurrence of plastic deformation can be interpreted to be due to the beginning of slidings in the areas of the basic tangential stresses. Also, the modeling of the plastic deformation mechanism in terms of the concept of sliding is efficient in formulating the strengthening laws [3]. In complex loading of anisotropic materials the peculiar manifestation of the Baushinger effect, observed, for example, in testing sample No. 6611, should be taken into account.

4. The complex loading of sample No. 6611 (Fig. 3) was performed in four steps: 1) axial tension up to the tension $\sigma_z^* = 28.076 \cdot 9.81$ MPa, 2) biaxial tension at $\sigma_z =$ const such that, in additional loading by the tension σ_φ, its final value became larger than σ_z^* by approximately a factor of 1.8; 3) unloading σ_φ ($\sigma_z = \sigma_z^*$), 4) resumption of axial tension ($\sigma_z > \sigma_z^*$, $\sigma_\varphi = 0$).

At the end of the first loading step, Γ_z=0.839% and $\Gamma_\varphi = -0.615$% (point A in Fig. 3, open circles). A comparison with the data of sample No. 6604 ($k_\sigma = \infty$, closed circles) shows that the strengthening for sample No. 6611 in the section of uniaxial tension is larger than the strengthening of the previous sample. A coefficient of reduction to the nominal diagram k_τ is introduced [3] in such cases in the processing of experimental results, so that the tension of a particular sample multiplied by the coefficient k_τ and the tension of a sample taken as the nominal one approximately coincide in the same plastic deformation. Such coincidence of this section the strengthening section of sample No. 6611 with the tension diagram of sample No. 6604 (which will be

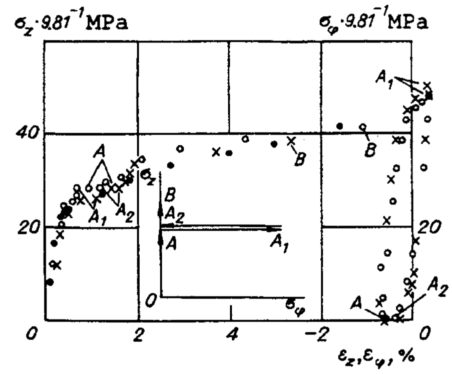

Fig. 3

considered nominal) is achieved if $k_r \approx 0.95$. In other words, to compare the calculated and experimental data of sample No. 6611, the tensions appearing in it, including those at the sections of complex loading, must be reduced by a factor of 0.95.

At the beginning of the second loading step, the strain component ε_z (and its constituent Γ_z) continued to increase, and ε_φ (and Γ_φ) decreased (Table 2); at the tension $\sigma_\varphi = 4.8 \cdot 9.81$ MPa ($\sigma_z = \sigma_z^*$), $\Gamma_z = 0.9506\%$, $\Gamma_\varphi = -0.6735\%$. This means that the slidings that took place in axial tension continued, but, apparently, with weaker intensity.

Further growth of the tension σ_φ, beginning with $\sigma_\varphi \approx 40 \cdot 9.81$ MPa, leads to a prevailing positive increment of the component Γ_φ, which is caused by slidings in the area $T_{\varphi r}$.

At the end of the second loading step, $\Gamma_z = 0.6235\%$, $\Gamma_\varphi = 0.002\%$ (in Fig. 3, point A_1, open circles, $\sigma_z = \sigma_z^*$ as before), the relation between the components Γ_z and Γ_φ changes in such a way that it indicates the "engagement" of the section T_{zr} even before complete relief of the tension σ_φ. After unloading, $\Gamma_z = 1.0863\%$, $\Gamma_\varphi = -0.6735\%$ (points A_2 in Fig. 3).

In the final step of loading, when the axial tension was continued, the diagram $\sigma_z(\varepsilon_z)$ taking into account the correction coefficient k_r approximates an analogous diagram of the proportional axial tension (closed circles in Fig. 3).

Sample No. 6631 was tested by the same loading program, as sample No. 6611. The strain variation pattern of this sample is similar to the pattern just considered (crosses in Fig. 3).

Thus, the behavior of not only isotropic materials, but also initially anisotropic materials, under loading can be explained in terms of the concept of sliding, which reflects the basic mechanism of the plasticity phenomenon of polycrystalline materials. Moreover, if we take into account that local slidings are concentrated mainly in the areas of basic tangential stresses, it becomes clear why the majority of constructional materials are orthotropic: owing to these slidings, this symmetry is maintained at any intensity of slidings in different basic areas T_{ij} of sliding.

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